Lecture 12: The Unit Cell and the Neutron Life Cycle

CBE 30235: Introduction to Nuclear Engineering — D. T. Leighton

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Reading Assignment

Lamarsh & Baratta (3rd or 4th Edition):

- **Section 6.5:** The Four-Factor Formula (The "Engine" of the reactor).
- **Section 6.8:** Heterogeneous Reactors (Why we lump fuel into rods).

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1 Introduction

We have discussed cross-sections (σ) for individual nuclei. Now we must build a machine. Real reactors are not homogeneous mixtures of atoms; they are repeating geometric structures called **Lattices**.

Today we define the "Unit Cell"—the smallest repeating volume of fuel and moderator—and trace the life cycle of a neutron within it. This leads to the fundamental figure of merit for the lattice: the **Infinite Multiplication Factor (k_∞)**.

2 The Neutron Life Cycle (The Four Factor Formula)

Imagine an infinite array of these unit cells. We track a generation of n thermal neutrons absorbed in the fuel.

2.1 1. Fission Production (η)

The cycle begins when thermal neutrons are absorbed by the fuel.

- Not every absorption causes fission (some are captured by ^{238}U).
- η **(Eta):** The average number of fission neutrons produced per *thermal neutron absorbed in the fuel.*

$$n \xrightarrow{\text{Fuel Absorption}} n\eta \quad \text{(Fast Neutrons Born)}$$

2.2 2. Fast Fission (ϵ)

The neutrons are born at high energy (2 MeV). Before they leave the fuel rod, they may strike a ^{238}U nucleus and cause a "fast fission" event.

- This is a small bonus effect.
- ϵ **(Epsilon):** The Fast Fission Factor (> 1). Typically ∼ 1.03.

$$n\eta \xrightarrow{\text{Fast Fission}} n\eta\epsilon$$

2.3 3. Slowing Down & Resonance Escape (p)

The neutrons leave the rod and enter the moderator (water). They must slow down from 2 MeV to 0.025 eV.

- **The Danger Zone:** As they pass through intermediate energies (1–1000 eV), they face the giant resonance peaks of ^{238}U. If they touch fuel at this energy, they die.
- p**:** The Resonance Escape Probability. (The chance of surviving the slow-down).

$$n\eta\epsilon \xrightarrow{\text{Slowing Down}} n\eta\epsilon p \quad \text{(Thermal Neutrons Reached)}$$

2.4 4. Thermal Utilization (f)

Now the neutrons are thermal. They diffuse through the lattice. They can be absorbed by the fuel (good) or by the moderator/cladding (bad).

- f**:** The Thermal Utilization Factor.

$$f = \frac{\text{Neutrons Absorbed in Fuel}}{\text{Total Neutrons Absorbed}}$$

$$n\eta\epsilon p \xrightarrow{\text{Diffusion}} n\eta\epsilon p f \quad \text{(New Generation)}$$

2.5 The Result

The ratio of the new generation to the old is:

$$k_\infty = \epsilon p f \eta \tag{1}$$

3 The "Rod" Problem: Why Heterogeneous?

Why do we construct reactors with fuel rods? Why not just dissolve uranium salts in water (Homogeneous)?

The Physics of Lumping:

1. **Resonance Protection (Improving p):** ^{238}U is incredibly "hungry" for neutrons at resonance energies. If you mix U and H atoms uniformly, the neutrons never escape. By lumping the fuel into rods, we force the neutrons to slow down in the *water*, physically separated from the ^{238}U.
2. **Thermal Disadvantage (Hurting f):** Lumping fuel makes it harder for *thermal* neutrons to get back in. The outer skin of the rod "shields" the inside.

4 Design Insight: How Big Should the Rod Be?

A student asked: *"What is the optimal radius for a fuel rod?"* This is a "Goldilocks" problem determined by the Mean Free Path of the neutron.

Quick Recall: The Mean Free Path

Before we optimize the rod size, recall from our discussion on cross-sections that the average distance a neutron travels before colliding with a nucleus is:

$$\lambda = \frac{1}{\Sigma_t} \tag{2}$$

where Σ_t is the macroscopic total cross-section.

- **In Fuel (Fast Neutrons):** Σ_t is small ($\lambda \approx 6$ cm).
- **In Fuel (Thermal Neutrons):** Σ_t is large ($\lambda \approx 1.5$ cm).

The Optimization Constraints

- **Constraint 1: Fast Neutron Escape ($\lambda_{fast} \approx 6$ cm)** Fission neutrons are born fast (2 MeV). They need to escape the rod to reach the water. Since $\lambda_{fast} \approx 6$ cm, a standard 1 cm rod is transparent to them. They fly out easily.
- **Constraint 2: Thermal Neutron Penetration ($\lambda_{thermal} \approx 1 - 2$ cm)** Thermal neutrons must diffuse back *into* the rod to cause fission. Fuel is very absorbent. If the rod is too thick (e.g., 6 cm), the neutrons will all be eaten on the surface, and the center will generate no power.
- **Constraint 3: The Thermal Limit (Melting)** Uranium Oxide (ceramic) is a poor heat conductor. If the rod is too thick, the heat cannot escape, and the centerline will melt ($T_{melt} \approx 2800°$C).

The Solution: Commercial fuel rods are almost universally $\sim$ **1** cm in diameter.

- Small enough for heat and thermal neutrons to get out/in.
- Large enough to provide structural integrity and fuel volume.

5 Historical Sidebar: The Natural Reactors of Oklo

You might think that a nuclear reactor requires precise engineering to separate fuel and moderator. In 1972, French physicists discovered that nature beat us to it by 1.7 billion years.

The Oklo Phenomenon (Gabon, Africa): Mining engineers found uranium ore that was depleted in ^{235}U (found at $\sim 0.4 - 0.6\%$ instead of the standard 0.72%). They traced it back to a deposit that had acted as a natural 100 kW nuclear reactor for hundreds of thousands of years.[1]

How did it work?

- **Higher Enrichment:** 1.7 billion years ago, the natural abundance of ^{235}U was $\sim 3\%$ (because it decays faster than ^{238}U). This is the same enrichment we use in modern PWRs!

[1] For more information, read the Wikipedia article: Natural nuclear fission reactor

- **Separation of Phases:** The uranium ore was concentrated in veins, and groundwater flooded the cracks and pores in the rock.
- **The "Lattice":** The rock provided the fuel; the water in the cracks provided the moderator. The heterogeneous separation allowed p to be high enough for criticality.
- **Passive Safety:** When the reactor got too hot, it boiled the water away. Without the moderator, the reaction stopped (negative void coefficient). When the rock cooled, water returned, and the reactor restarted.

Note: The Oklo reactors are also cited in waste disposal studies because the fission products (like Xenon and Neodymium) largely remained trapped in the rock for 2 billion years, moving only centimeters.