

# Lecture 12: The Unit Cell and the Neutron Life Cycle

CBE 30235: Introduction to Nuclear Engineering — D. T. Leighton

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## Reading Assignment

Lamarche & Baratta (3rd or 4th Edition):

- **Section 6.5:** The Four-Factor Formula (The "Engine" of the reactor).
- **Section 6.8:** Heterogeneous Reactors (Why we lump fuel into rods).

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## 1 Introduction

We have discussed cross-sections ( $\sigma$ ) for individual nuclei. Now we must build a machine. Real reactors are not homogeneous mixtures of atoms; they are repeating geometric structures called **\*\*Lattices\*\***.

Today we define the "Unit Cell"—the smallest repeating volume of fuel and moderator—and trace the life cycle of a neutron within it. This leads to the fundamental figure of merit for the lattice: the **\*\*Infinite Multiplication Factor ( $k_\infty$ )\*\***.

## 2 The Neutron Life Cycle (The Four Factor Formula)

Imagine an infinite array of these unit cells. We track a generation of  $n$  thermal neutrons absorbed in the fuel.

### 2.1 1. Fission Production ( $\eta$ )

The cycle begins when thermal neutrons are absorbed by the fuel.

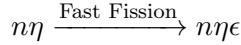
- Not every absorption causes fission (some are captured by  $^{238}\text{U}$ ).
- $\eta$  (**Eta**): The average number of fission neutrons produced per *thermal neutron absorbed in the fuel*.

$$n \xrightarrow{\text{Fuel Absorption}} n\eta \quad (\text{Fast Neutrons Born})$$

## 2.2 2. Fast Fission ( $\epsilon$ )

The neutrons are born at high energy (2 MeV). Before they leave the fuel rod, they may strike a  $^{238}\text{U}$  nucleus and cause a "fast fission" event.

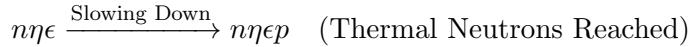
- This is a small bonus effect.
- $\epsilon$  (**Epsilon**): The Fast Fission Factor ( $> 1$ ). Typically  $\sim 1.03$ .



## 2.3 3. Slowing Down & Resonance Escape ( $p$ )

The neutrons leave the rod and enter the moderator (water). They must slow down from 2 MeV to 0.025 eV.

- **The Danger Zone:** As they pass through intermediate energies (1–1000 eV), they face the giant resonance peaks of  $^{238}\text{U}$ . If they touch fuel at this energy, they die.
- $p$ : The Resonance Escape Probability. (The chance of surviving the slow-down).



## 2.4 4. Thermal Utilization ( $f$ )

Now the neutrons are thermal. They diffuse through the lattice. They can be absorbed by the fuel (good) or by the moderator/cladding (bad).

- $f$ : The Thermal Utilization Factor.

$$f = \frac{\text{Neutrons Absorbed in Fuel}}{\text{Total Neutrons Absorbed}}$$



## 2.5 The Result

The ratio of the new generation to the old is:

$$k_\infty = \epsilon p f \eta \quad (1)$$

## 3 The "Rod" Problem: Why Heterogeneous?

Why do we construct reactors with fuel rods? Why not just dissolve uranium salts in water (Homogeneous)?

### The Physics of Lumping:

1. **Resonance Protection (Improving  $p$ ):**  $^{238}\text{U}$  is incredibly "hungry" for neutrons at resonance energies. If you mix U and H atoms uniformly, the neutrons never escape. By lumping the fuel into rods, we force the neutrons to slow down in the *water*, physically separated from the  $^{238}\text{U}$ .
2. **Thermal Disadvantage (Hurting  $f$ ):** Lumping fuel makes it harder for *thermal* neutrons to get back in. The outer skin of the rod "shields" the inside.

## 4 Design Insight: How Big Should the Rod Be?

A student asked: *”What is the optimal radius for a fuel rod?”* This is a ”Goldilocks” problem determined by the Mean Free Path of the neutron.

### Quick Recall: The Mean Free Path

Before we optimize the rod size, recall from our discussion on cross-sections that the average distance a neutron travels before colliding with a nucleus is:

$$\lambda = \frac{1}{\Sigma_t} \quad (2)$$

where  $\Sigma_t$  is the macroscopic total cross-section.

- **In Fuel (Fast Neutrons):**  $\Sigma_t$  is small ( $\lambda \approx 6$  cm).
- **In Fuel (Thermal Neutrons):**  $\Sigma_t$  is large ( $\lambda \approx 1.5$  cm).

### The Optimization Constraints

- **Constraint 1: Fast Neutron Escape ( $\lambda_{fast} \approx 6$  cm)** Fission neutrons are born fast (2 MeV). They need to escape the rod to reach the water. Since  $\lambda_{fast} \approx 6$  cm, a standard 1 cm rod is transparent to them. They fly out easily.
- **Constraint 2: Thermal Neutron Penetration ( $\lambda_{thermal} \approx 1 - 2$  cm)** Thermal neutrons must diffuse back *into* the rod to cause fission. Fuel is very absorbent. If the rod is too thick (e.g., 6 cm), the neutrons will all be eaten on the surface, and the center will generate no power.
- **Constraint 3: The Thermal Limit (Melting)** Uranium Oxide (ceramic) is a poor heat conductor. If the rod is too thick, the heat cannot escape, and the centerline will melt ( $T_{melt} \approx 2800^\circ\text{C}$ ).

**The Solution:** Commercial fuel rods are almost universally  $\sim 1$  cm in diameter.

- Small enough for heat and thermal neutrons to get out/in.
- Large enough to provide structural integrity and fuel volume.

## 5 Historical Sidebar: The Natural Reactors of Oklo

You might think that a nuclear reactor requires precise engineering to separate fuel and moderator. In 1972, French physicists discovered that nature beat us to it by 1.7 billion years.

**The Oklo Phenomenon (Gabon, Africa):** Mining engineers found uranium ore that was depleted in  $^{235}\text{U}$  (found at  $\sim 0.4 - 0.6\%$  instead of the standard  $0.72\%$ ). They traced it back to a deposit that had acted as a natural 100 kW nuclear reactor for hundreds of thousands of years.<sup>1</sup>

### How did it work?

- **Higher Enrichment:** 1.7 billion years ago, the natural abundance of  $^{235}\text{U}$  was  $\sim 3\%$  (because it decays faster than  $^{238}\text{U}$ ). This is the same enrichment we use in modern PWRs!

<sup>1</sup>For more information, read the Wikipedia article: [Natural nuclear fission reactor](#)

- **Separation of Phases:** The uranium ore was concentrated in veins, and groundwater flooded the cracks and pores in the rock.
- **The "Lattice":** The rock provided the fuel; the water in the cracks provided the moderator. The heterogeneous separation allowed  $p$  to be high enough for criticality.
- **Passive Safety:** When the reactor got too hot, it boiled the water away. Without the moderator, the reaction stopped (negative void coefficient). When the rock cooled, water returned, and the reactor restarted.

*Note: The Oklo reactors are also cited in waste disposal studies because the fission products (like Xenon and Neodymium) largely remained trapped in the rock for 2 billion years, moving only centimeters.*